1. Express the following in terms of j:

(a) $\sqrt{4-7}$ (b) $\sqrt{-144}$

(c) $\dfrac{\sqrt{5}}{\sqrt{-4}}$ (d) $\sqrt{4(-25)}$

(a) $\sqrt{4-7} = \sqrt{-3} = \sqrt{3}\sqrt{-1} = \sqrt{3}\,j$

(b) $\sqrt{-144} = |12|\,j = 12j$

(c) $\dfrac{\sqrt{5}}{\sqrt{-4}} = \dfrac{\sqrt{5}}{2j} \cdot \dfrac{j}{j} = -\dfrac{\sqrt{5}}{2}\,j$

(d) $\sqrt{4\cdot(-25)} = \sqrt{4}\sqrt{-25} = 2\cdot 5j = 10j$

2. Compute

(a) j^8 (b) j^{15}

(c) j^{45} (d) $(-j)^3$

(a) $j^8 = j^{4\cdot 2} = (j^4)^2 = j^2 = 1$

(b) $j^{15} = j^{12+3} = j^{4\cdot 3}\cdot j^3 = 1\cdot j^3 = 1\cdot(-j) = -j$

(c) $j^{45} = j^{15\cdot 3} = (-j)^3 = j$

(d) $(-j)^3 = j$

3. Evaluate

(a) $\sqrt{-48}+\sqrt{-75}-\sqrt{-27}$

(b) $\sqrt{-12}-\sqrt{-8}+\sqrt{-0.6}$

(c) $\sqrt{-3}\sqrt{-3}$

(d) $\sqrt{-a}\sqrt{+b}$

(e) $5\mathrm{j}^3 2\mathrm{j}^6$

(f) $(-\mathrm{j})^3\mathrm{j}^2$

(g) $8\mathrm{j}/2\mathrm{j}$

(h) $1/j^3$

(i) $6\mathrm{j}/\mathrm{j}^7\sqrt{3}$

(j) $1/\mathrm{j}^5+1/\mathrm{j}^7$

(k) $\sqrt{b-a}\sqrt{a-b}$

(l) $\dfrac{\sqrt{-3}\sqrt{12}}{\mathrm{j}\sqrt{-a^2}}$

(a) $\sqrt{-48}+\sqrt{-75}+\sqrt{-27} = \sqrt{-16\cdot 3}+\sqrt{-25\cdot 3}+\sqrt{-9\cdot 3}$

$= 4i\sqrt{3}+5i\sqrt{3}+3i\sqrt{3}$

$= 12i\sqrt{3}$

(b) $\sqrt{-12}-\sqrt{-8}+\sqrt{-0.6} = 2\sqrt{3}\,j - 2\sqrt{2}\,j + \sqrt{\frac{5}{3}}\,j$

(c) $\sqrt{-3}\sqrt{-3} = \sqrt{3}j\cdot\sqrt{3}j = -3$

(d) $\sqrt{-a}\sqrt{+b} = \sqrt{ab}\,j$

(e) $5j^3 2j^6 = 5(-j)\cdot 2\cdot(-1) = 10j$

(f) $(-j)^3 j^2 = -j$

(g) $\frac{8j}{2j} = 4$

(h) $\frac{1}{j^3} = \frac{1}{-j} = j$

(i) $\frac{6j}{j^7}\sqrt{3} = 6\frac{1}{j^6}\sqrt{3} = -6\sqrt{3}$

(j) $\frac{1}{j^5} + \frac{1}{j^7} = \frac{1}{j^1} + \frac{1}{j^3} = \frac{1}{j} + \frac{1}{-j} = 0$

(k) $\sqrt{b-a}\,\sqrt{a-b} = \sqrt{(b-a)(a-b)}$

$= \sqrt{ab - b^2 - a^2 + ab}$

$= \sqrt{2ab - b^2 - a^2}$

$= \sqrt{(-1)(a^2 - 2ab + b^2)}$

$= j\sqrt{(a-b)^2}$

$= j(a-b)$

(l) $\frac{\sqrt{-3}\,\sqrt{12}}{j\sqrt{-a^2}} = \frac{j\sqrt{3}\,2\sqrt{3}}{j \cdot j\sqrt{a^2}} = \frac{2 \cdot 3}{j|a|}\,\frac{j}{j} = \pm\frac{6}{a}j$

4. Determine the imaginary part of z:

   (a) $z = 3 + 7j$
   (b) $z = 15j - 4$

(a) $\mathrm{Im}(z) = 7$ | (b) $\mathrm{Im}(z) = 15$

5. Determine the conjugate complex number z^* of z:

   (a) $z = 5 + 2j$
   (b) $z = \frac{1}{2} - \sqrt{3}j$

(a) $z^* = 5 - 2j$ | (b) $z^* = \frac{1}{2} + \sqrt{3}j$

6. Evaluate the (complex) roots of the following quadratic equations:

(a) $x^2 + 4x + 13 = 0$

(b) $x^2 + \frac{3}{2}x + \frac{25}{16} = 0$

(a) $x^2 + 4x + 13 = 0$

$$\Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 13}}{2 \cdot 1}$$

$$= -2 + 3j \;/\; -2 - 3j$$

(b) $x^2 + \frac{3}{2}x + \frac{25}{16} = 0$

$$\Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{3}{2} \pm \sqrt{\frac{9}{4} + 4 \cdot 1 \cdot \frac{25}{16}}}{2}$$

$$= -\frac{3}{4} + i \;/\; -\frac{3}{4} - i$$

7. Calculate the sum $z_1 + z_2$:

(a) $z_1 = 3 - 2j$

$z_2 = 7 + 5j$

(b) $z_1 = \frac{3}{4} + \frac{3}{4}j$

$z_2 = \frac{3}{4} - \frac{3}{4}j$

(a) $z_1 + z_2 = 3 - 2j + 7 + 5j$

$= 10 + 3j$

$z_1 + z_2 = \frac{3}{4} + \frac{3}{4}j + \frac{3}{4} - \frac{3}{4}j$

$= \frac{3}{2}$

8. Compute $w = z_1 - z_2 + z_3^*$:

(a) $z_1 = 5 - 2\text{j}$
$z_2 = 2 - 3\text{j}$
$z_3 = -4 + 6\text{j}$

(b) $z_1 = 4 - 3.5\text{j}$
$z_2 = 3 + 2\text{j}$
$z_3 = 7.5\text{j}$

(a) $w = z_1 - z_2 + z_3^*$
$= 5 - 2j - (2 - 3j) + (-4 - 6j)$
$= 5 - 2j - 2 + 3j - 4 - 6j$
$= -1 - 5j$

$w = z_1 - z_2 + z_3^*$
$= 4 - \frac{7}{2}j - (3 + 2j) + (-\frac{15}{2}j)$
$= 4 - \frac{7}{2}j - 3 - 2j - \frac{15}{2}j$
$= 1 - 13j$

9. Compute the product $w = z_1 z_2$:

(a) $z_1 = 1 + \text{j}$
$z_2 = 1 - \text{j}$

(b) $z_1 = 3 - 2\text{j}$
$z_2 = 5 + 4\text{j}$

(a) $w = z_1 z_2$
$= (1 + j)(1 - j)$
$= 1 - j + j + 1$
$= 2$

(b) $w = z_1 \cdot z_2$
$= (3 - 2j)(5 + 4j)$
$= 15 + 12j - 10j + 8$
$= 23 + 2j$

10. Determine

(a) $(16 + \text{j}\sqrt{2})/2\sqrt{2}$

(b) $(4 - j\sqrt{3})/2\text{j}$

(c) $(2 + 3\text{j})/(2 - 4\text{j})$

(d) $1/(1 + \text{j})$

(e) $\dfrac{1 + \text{j}}{1 - \text{j}} - \dfrac{1 - \text{j}}{1 + \text{j}}$

(f) $\dfrac{(5 + j\sqrt{3})(5 - \text{j}\sqrt{3})}{2 - \text{j}\sqrt{3}}$

(a) $\frac{16+j\sqrt{2}}{2\sqrt{2}} = \frac{16}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}}j$

$= \frac{8\sqrt{2}}{\sqrt{2}\sqrt{2}} + \frac{1}{2}j$

$= 4\sqrt{2} + \frac{1}{2}j$

(b) $\frac{4-j\sqrt{3}}{2j} = \frac{4}{2j} - \frac{\sqrt{3}j}{2j}$

$= -2j - \frac{\sqrt{3}}{2}$

(c) $\frac{2+3j}{2-4j} \cdot \frac{2+4j}{2+4j} = \frac{(2+3j)(2+4j)}{2^2+4^2}$

$= \frac{4+8j+6j-12}{20}$

$= -\frac{4}{10} + \frac{7}{10}j$

$= -\frac{2}{5} + \frac{7}{10}j$

(d) $\frac{1}{1+j} = \frac{1-j}{2}$

$= \frac{1}{2} - \frac{1}{2}j$

(e) $\frac{1+j}{1-j} - \frac{1-j}{1+j} = \frac{(1+j)(1+j)-(1-j)(1-j)}{(1-j)(1+j)}$

$= \frac{1+2j-1-(1-j-j-1)}{1+j-j+1}$

$= \frac{4j}{2}$

$= 2j$

(f) $\dfrac{(5+j\sqrt{3})(5-j\sqrt{3})}{2-j\sqrt{3}}$ $\quad \dfrac{25 - j5\sqrt{3} + j5\sqrt{3} + 3}{2-j\sqrt{3}}$

$= \dfrac{28}{2-j\sqrt{3}} \cdot \dfrac{2+j\sqrt{3}}{2+j\sqrt{3}}$

$= \dfrac{28(2+j\sqrt{3})}{2^2+3}$

$= \dfrac{56 + 28j\sqrt{3}}{7}$

$= 8 + 4j\sqrt{3}$

11. Convert the following sums into products:

(a) $4x^2 + 9y^2$ (b) $a + b$

(a) $4x^2 + 9y^2 = (2x + 3jy)(2x - 3jy)$

(b) $a + b = (\sqrt{a} + j\sqrt{b})(\sqrt{a} - j\sqrt{b})$